Standardization of Ice-Cream Mixes

By P. H. TRACY

UNIVERSITY OF ILLINOIS
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By P. H. TRACY, Assistant Chief in Dairy Manufactures

By ice-cream standardization is meant the mathematical procedure necessary to determine the amounts of the various ingredients needed to make a product of the desired weight and composition. Correct standardization is essential in the production of ice cream that is uniform in composition. Since variations in composition may result in poor quality, and since ice-cream solids are expensive, it is important from an economic standpoint to secure a product uniform in composition. The fact that nearly all the states have laws regulating the fat and in some cases the content of serum solids in ice cream also makes it imperative that the manufacturer understand how to standardize his ice-cream mixes.

There are three types of standardization problems: (1) those encountered when it is desired to use milk or cream of a different test than that called for in the mix formula; (2) those encountered when it is necessary to calculate all the ingredients needed to make a mix of a definite composition; and (3) those which are met when it is necessary to correct either the wrong proportion or the wrong concentration of solids in the mix.

Before proceeding with an explanation of these three types of problems consideration will be given to certain fundamentals a knowledge of which is necessary for a proper understanding of ice-cream standardization, namely the composition of milk products, milk and cream standardization, and the calculations of the solids in a mix from a given formula.

COMPOSITION OF MILK PRODUCTS

Composition of Milk

Milk is composed of butterfat, serum solids (sometimes referred to as milk-solids-not-fat), and water. The amount of each of these three classes of ingredients varies but, on the average, milk may be said to contain 4 percent fat, 9 percent serum solids, and 87 percent water. The serum solids consist of milk sugar, casein, albumin, and ash and, together with the water in the milk, are called *serum*.

Composition of Cream

Cream is a mixture of serum and butterfat. The difference between cream and milk is in the proportion of fat to serum. As

the amount of fat in cream increases, the amount of serum decreases; therefore, the higher the fat percentage in cream, the lower the percentage of serum solids.

The composition of the serum in cream will vary somewhat, depending probably upon the composition of the milk from which it is separated and the method by which the cream is removed from the milk. However, if the average composition of the milk used in the plant is known, the average percentage of solids in the serum can be calculated, and a value determined that is satisfactory for practical purposes. For instance, if the average test of the milk received at the plant is 4 percent fat and 8.8 percent serum solids, the percentage solids in the serum of the cream separated from this milk can be calculated as in the following example:

First, it is assumed that the process of separating cream from milk does not affect the distribution of the solids in the serum.[1] If the fat is removed from 100 pounds of the whole milk, there remains 96 pounds of serum containing 8.8 pounds of solids. Therefore, the percentage of solids in the 96 pounds of serum would be equal to $\frac{8.8}{96} \times 100 = 9.166$. After thus arriving at a value for the percentage of solids in the serum, the percentage of serum solids in the cream can be calculated by the following formula:

(100 − percentage fat in cream) × percentage solids in serum

As an illustration we may calculate the percentage of serum solids in 40-percent cream, assuming that the percentage solids in the serum is 9.2.[2] We then have:

$100 - 40 = 60 \qquad 60 \times .092 = 5.52$ percent

Composition of Condensed Milk Products

To obtain an ice cream sufficiently high in serum solids, it is necessary to introduce into the mix a milk product in which the serum solids have been concentrated thru the removal of water. For this purpose either sweetened or unsweetened condensed whole, skim, or partly skimmed milk, or skim-milk powder may be used. The more common of these products are included in Table 1.

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[1]Sanmann, F. P., and Overman, O. R. Relation of composition of skim and full milk. Creamery and Milk Plant Monthly. April, 1927.

[2]This value will be used thruout this publication as the percentage of solids in normal serum and is written .092 when expressed decimally.

TABLE 1.—PERCENTAGE COMPOSITION AND WEIGHTS PER GALLON OF COMMON INGREDIENTS USED IN ICE-CREAM MANUFACTURE

Ingredient	Fat	Serum solids	Total solids	Weight per gallon[1]
	perct.	*perct.*	*perct.*	*lbs.*
Water	.....	.....	.....	8.32
Skim milk	.06	9.24	9.30	8.63
Milk	4.00	9.00	13.00	8.59
Cream	18.00	7.54	25.54	8.43
Cream	20.00	7.36	27.36	8.45
Cream	25.00	6.90	31.90	8.39
Cream	30.00	6.44	36.44	8.35
Cream	35.00	5.98	40.98	8.31
Cream	40.00	5.52	45.52	8.28
Cream	45.00	5.06	50.06	8.24
Cream	50.00	4.60	54.60	8.21
Butter	83.00	1.00	84.00	....
Condensed skim milk	.....	.....	20.00	8.98
Condensed skim milk	.....	.....	25.00	9.18
Condensed skim milk	.....	.....	30.00	9.35
Condensed skim milk	.....	.....	35.00	9.53
Condensed whole milk	8.00	22.00	30.00	8.99
Condensed whole milk	8.00	25.00	33.00	9.09
Sweetened condensed skim milk	.50	27.50	70.00	11.18
Sweetened condensed whole milk	8.00	20.00	72.50	10.87
Evaporated milk	7.80	17.70	25.50	8.88
Skim-milk powder	1.35	95.85	97.20	6.00
Ice-cream mix[2]	9.64	15.56	25.20	8.78
Ice-cream mix[2]	12.05	12.05	24.10	8.64
Ice-cream mix[2]	14.46	10.84	25.30	8.61
Ice-cream mix[2]	14.46	14.46	28.92	8.71
Ice-cream mix[2]	16.87	12.05	28.92	8.61
Ice-cream mix[2]	16.87	14.46	31.33	8.69
Ice-cream mix[3]	8.00	8.00	29.50	9.04
Ice-cream mix[3]	8.00	14.00	35.50	9.19
Ice-cream mix[3]	14.00	8.00	35.50	8.97
Ice-cream mix[3]	10.00	12.00	35.50	9.15
Ice-cream mix[3]	8.50	12.00	34.00	9.17
Ice-cream mix[3]	12.00	10.00	35.50	9.07
Ice-cream mix[3]	16.00	8.50	38.00	8.98
Vanilla extract (60 percent alcohol)	.....	.....	.....	7.87
Vanilla extract (50 percent alcohol)	.....	.....	.....	7.93
Vanilla extract (40 percent alcohol)	.....	.....	.....	8.12
Granulated sugar	.....	.....	99.90	7.19
Granulated gelatin	.....	.....	90.00	4.66

[1]At 60° F.

[2]Calculated to be 83 percent of the finished mix. The 17 percent which remains to be added is sugar and gelatin solution.

[3]13 percent sugar and .5 percent gelatin.

MILK AND CREAM STANDARDIZATION

Pearson's Square Method

This method, which is most commonly used, proceeds as follows:

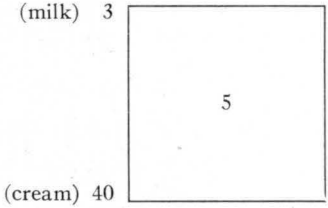
Problem 1

How much 3-percent milk and 40-percent cream are needed to make 500 pounds of 5-percent milk?

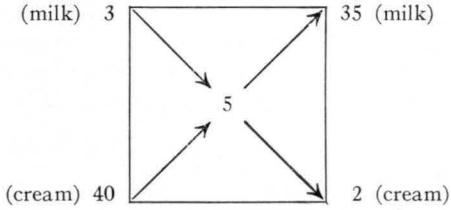
Solution

(1) Draw a square and place in the center the percentage of fat desired:

(2) Place at the left-hand corners of the square the percentages of fat in the materials to be mixed:



(3) Subtract the number in the center of the square from the larger number on the left, and place the result at the corner diagonally opposite. Then subtract the smaller number on the left from the number in the center of the square, and place the result at the corner diagonally opposite:



(4) The two numbers at the right-hand corners of the square represent the pounds (or parts by weight) of the two products required.

In this case we find that 35 parts of 3-percent milk, when added to 2 parts of 40-percent cream, will make 5-percent milk.

(5) Since 35 parts of 3-percent milk when mixed with 2 parts of 40-percent cream will make 5-percent milk, then 35 pounds of 3-percent milk when added to 2 pounds of 40-percent cream will make 37 pounds of 5-percent milk. However, since it is desired to make 500 pounds of 5-percent milk, the amounts of 3-percent milk and 40-percent cream used must be increased proportionately:

Let x represent the pounds of cream needed
Then $2 : 37 = x : 500$
$37x = 1000$
$x = 27.02$ pounds of 40-percent cream needed
$500 - 27.02 = 472.98$ pounds of 3-percent milk needed

Proof

Five hundred pounds of 5-percent milk contains 25 pounds of fat. Therefore, the amount of fat in the calculated amounts of 40-percent cream and 3-percent milk should total 25 pounds:

$27.02 \times .40 = 10.8080$ pounds fat in cream
$472.98 \times .03 = 14.1894$ pounds fat in milk
24.9974 (25) pounds fat in mixture

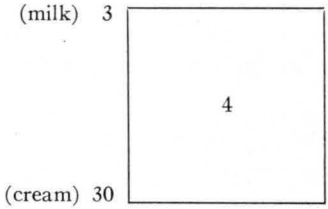
Problem 2

How much 30-percent cream will be needed to standardize 260 pounds of 3-percent milk so that the fat content will be 4 percent?

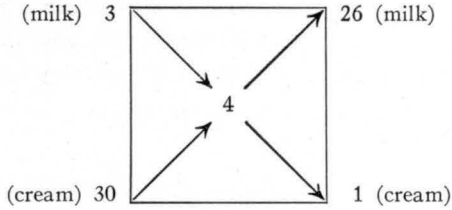
Solution

(1) Place in the center of a square the percentage of fat desired:

(2) Place at the left-hand corners of the square the percentages of fat in the materials to be mixed:



(3) Subtract the number in the center of the square from the larger number on the left, and place the result at the corner diagonally opposite. Then subtract the smaller number on the left from the number in the center of the square, and place the result at the corner diagonally opposite:



(4) The two numbers at the right-hand corners of the square represent the parts by weight of the two products that are needed: that is, 26 parts of 3-percent milk, when added to 1 part of 30-percent cream, will make 4-percent milk.

(5) Since 26 parts of 3-percent milk when mixed with 1 part of 40-percent cream will make 4-percent milk, then 26 pounds of 3-percent milk when added to 1 pound of 30-percent cream will make 27 pounds of 4-percent milk. However, there are 260 pounds of 3-percent milk to be mixed with 30-percent cream so as to make 4-percent milk. The amount of 30-percent cream needed is calculated as follows:

$$1 : 26 = x : 260$$
$$26x = 260$$
$$x = 10 \text{ pounds of 30-percent cream needed}$$

Proof

The weight of the standardized 4-percent milk is equal to (260 + 10) 270 pounds. Two hundred seventy pounds of 4-percent milk contains 10.8 pounds of fat. Therefore the amount of fat in the calculated amounts of 3-percent milk and 30-percent cream should total 10.8 pounds:

$$260 \times .03 = 7.8 \text{ pounds fat in milk}$$
$$10 \times .30 = 3.0 \text{ pounds fat in cream}$$
$$10.8 \text{ pounds fat in 270-pound mixture}$$

Formulas for Calculating Milk and Cream Standardization

The following formulas may be used in milk and cream standardization to determine the pounds of milk and cream needed for a certain desired result. The same problems are used here as were used to illustrate Pearson's Square Method.

Problem

How much 3-percent milk and 40-percent cream are needed to make 500 pounds of 5-percent milk?

Solution

$$\text{Cream} = \frac{\%\text{ fat desired} - \%\text{ fat in milk}}{\%\text{ fat in cream} - \%\text{ fat in milk}} \times \text{desired weight}$$

$$\text{Milk} = \frac{\%\text{ fat in cream} - \%\text{ fat desired}}{\%\text{ fat in cream} - \%\text{ fat in milk}} \times \text{desired weight}$$

$$\text{Cream} = \frac{5 - 3}{40 - 3} \times 500 = 27.02 \text{ pounds}$$

$$\text{Milk} = 500 - 27.02 = 472.98 \text{ pounds, or using formula}$$

$$\text{Milk} = \frac{40 - 5}{40 - 3} \times 500 = 472.973 \text{ pounds}$$

Problem

How much 30-percent cream will be needed to standardize 260 pounds of 3-percent milk to 4-percent?

Solution

$$260 = \frac{30 - 4}{30 - 3} \times \text{desired weight}$$

$$260 = \frac{26}{27} \times \text{desired weight}$$

$$26 \times \text{desired weight} = 260 \times 27$$

$$\text{Desired weight} = \frac{260 \times 27}{26} = 270 \text{ pounds}$$

$$270 - 260 = 10 \text{ pounds cream needed}$$

Proof (See pages 7 and 8)

CALCULATING THE SOLIDS IN A MIX

Occasions sometimes arise when it is necessary to know the percentage of each solid that would result from using a given mix formula. In order to calculate accurately the desired values, the exact composition of each of the milk products should be known In case this information is not obtainable, approximate results can be had by using the compositions given in Table 1.

The percentage of any solid present in the mix is equal to the total weight of the solid divided by the weight of the mix, multiplied by 100. In case the mix formula is expressed in volume, it must be converted to a weight basis before the calculations can be made.

Problem

We have the following mix formula and wish to calculate the percentage of each solid present:

	Pounds
2½ gals. 40-percent cream (5.5 percent serum solids)	20.0
5⅓ gals. 4-percent milk (8.9 percent serum solids)	46.0
2⅔ gals. 8-percent condensed milk (20 percent serum solids, 44 percent sugar)	28.0
½ pound gelatin (85 percent solids[1])	.5
½ pound vanilla	.5
3 pounds water for the gelatin	3.0
2 pounds sugar	2.0
Total weight	100.0

Solution

(1) To calculate the gelatin:

$$\frac{.85 \times .5}{100} \times 100 = .425 \text{ percent gelatin solids in mix}$$

(2) To calculate the sugar:

$$.44 \times 28 = 12.32 \text{ pounds sugar in condensed milk}$$

$$\frac{12.32 + 2}{100} = 14.32 \text{ percent sugar in mix}$$

(3) To calculate the fat:

$$20 \times .4 = 8.00 \text{ pounds fat in cream}$$
$$46 \times .04 = 1.84 \text{ pounds fat in milk}$$
$$28 \times .08 = 2.24 \text{ pounds fat in condensed milk}$$
$$12.08 \text{ pounds fat in mix}$$

$$\frac{12.08}{100} \times 100 = 12.08 \text{ percent fat in mix}$$

(4) To calculate the serum solids:

$$20 \times .055 = 1.10 \text{ pounds serum solids in cream}$$
$$46 \times .089 = 4.09 \text{ pounds serum solids in milk}$$
$$28 \times .20 = 5.60 \text{ pounds serum solids in condensed milk}$$
$$10.79 \text{ pounds serum solids in mix}$$

$$\frac{10.79}{100} \times 100 = 10.79 \text{ percent serum solids in mix}$$

(5) To calculate the total solids:

$$.425 \text{ percent gelatin solids}$$
$$14.32 \text{ percent sugar}$$
$$12.08 \text{ percent fat}$$
$$10.79 \text{ percent serum solids}$$
$$37.615 \text{ percent total solids in mix}$$

[1]The solids in gelatin vary from 80 to 90 percent altho gelatin is often considered as containing 100 percent solids.

STANDARDIZING THE MIX

I—WHEN MILK OR CREAM TO BE USED IS OF DIFFERENT TEST FROM THAT CALLED FOR IN FORMULA

This represents a very simple type of problem in which the necessary operations can be best explained by an example.

Problem

The mix formula is as follows:

6 gals. 18-percent cream
3 gals. 8-percent unsweetened condensed milk (25% serum solids)
1 gal. 4-percent milk
13 pounds sugar
5 pounds 10-percent gelatin solution

Eighteen-percent cream and 4-percent milk not being available, 40-percent cream and 3-percent milk must be used instead. Calculate the amount of 40-percent cream and 3-percent milk necessary to supply the desired fat in the proper bulk.

Solution

(1) To find the weight of the milk and cream mixture:

1 gallon of 18-percent cream weighs 8.45 pounds
1 gallon of 4-percent milk weighs 8.6 pounds

$$\begin{aligned} \text{Cream} &= 6 \times 8.45 = 50.7 \text{ pounds} \\ \text{Milk} &= 1 \times 8.6 = \underline{8.6 \text{ pounds}} \\ & \qquad 59.3 \text{ pounds} \end{aligned}$$

(2) To find the weight of the fat in the cream and milk mixture:

$$\begin{aligned} 50.7 \times .18 &= 9.126 \text{ pounds fat in cream} \\ 8.6 \times .04 &= \underline{.344 \text{ pounds fat in milk}} \\ & \qquad 9.470 \text{ pounds total fat} \end{aligned}$$

$$\frac{9.470}{59.3} \times 100 = 15.97 \text{ percent fat}$$

(3) Using the formula given under *Milk and Cream Standardization*, page 9, find the amount of cream and milk to be used:

$$\text{Cream} = \frac{15.97 - 3}{40 - 3} \times 59.3 = 20.79 \text{ pounds}$$

$$\text{Milk} = 59.3 - 20.79 = 38.51 \text{ pounds}$$

Proof

$$\begin{aligned} 20.79 \times .40 &= 8.316 \text{ pounds fat in cream} \\ 38.51 \times .03 &= \underline{1.155 \text{ pounds fat in milk}} \\ & \qquad 9.471 \text{ pounds total fat} \end{aligned}$$

II—CALCULATING INGREDIENTS FOR MIX OF CERTAIN COMPOSITION

In order to calculate the amounts of the different ingredients needed to make a given mix, one must know the weight and composition of the mix desired, its proportion of milk products, and the composition of the available milk products.

The problem may be solved in a number of different ways since in recent years several methods[1] have been developed. The three methods presented in this publication are the arithmetical (serum point), the algebraic, and the tabular.

Arithmetical Method[2]

Problem

Desired: 1000 pounds of mix containing 9 percent fat, 11 percent serum solids, 14 percent sugar, .5 percent gelatin, .5 percent vanilla.

Available: 30-percent cream, 4-percent milk, 8-percent condensed milk containing 20 percent serum solids and 40 percent sugar.

Solution

(1) To find the amounts of sugar, gelatin, and vanilla needed, multiply the percentage of each desired by 100:

$.14 \times 100 = 14$ pounds sugar (including that in condensed milk)
$.005 \times 100 = .5$ pound gelatin
$.005 \times 100 = .5$ pound vanilla

(2) To find the amount of condensed milk needed:

(a) Determine the pounds of serum[3] in 100 pounds of mix:

$$100 - (14 + .5 + .5 + 9) = 76$$

(b) Find the pounds of *normal*[4] serum solids in the serum of the mix:

$$76 \times .092 = 6.992$$

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[1]Cutler, T. D. Proportioning mixes made easy. Ice Cream Trade Jour. September, 1922. Doty, H. E. An arithmetical method of balancing the serum solids in ice cream mix. Ice Cream Rev. June, 1924. LaSalle, J. H. Proportioning mixes by arithmetic. Ice Cream Trade Jour. April, 1925. Sommer, H. H. A graphical method for figuring complex ice cream mixes. Ice Cream Rev. December, 1923. Fisk, W. W. The book of ice cream, pages 297-333. Macmillan, 1919.

[2]All calculations are on the basis of 100 pounds of mix.

[3]Serum = serum solids (milk-solids-not-fat) plus water.

[4]In uncondensed serum, the weight of the solids is 9.2 percent of the weight of the serum. This amount is therefore spoken of as *normal.* Subtracting this amount from the total serum solids of condensed milk or of a mix leaves an excess or *abnormal* percentage of serum solids, in this case $11[-(.092 \times 76)] = 4.008$ pounds.

(c) Find the amount of serum solids that must be added in a concentrated form by subtracting the result obtained in (b) from the total pounds of serum solids needed:

$.11 \times 100 = 11$ pounds serum solids needed
$11 - 6.992 = 4.008$

(d) Divide the result obtained in (c) by the percentage of serum solids in the condensed milk from which has been subtracted the percentage of *normal* serum solids in the condensed milk. The result multiplied by 100 equals the amount of condensed milk needed:

$$\frac{4.008}{20 - .092[100 - (8 + 40)]} \times 100 = \frac{4.008}{20 - 4.784} \times 100 = 26.34^{1}$$

(e) The different steps in calculating the pounds of condensed milk needed can be expressed as follows:

$$\text{Pounds condensed milk} = \frac{(\text{pounds s.s.}^{2}\text{ needed}) - (.092 \times \%\text{ serum in mix})}{(\%\text{ s.s. in condensed}) - (.092 \times \%\text{ serum in condensed})} \times 100$$

(3) To find the amount of milk and cream needed:

(a) Determine the pounds of milk products in 100 pounds of the mix:

$100 - [14\text{ (sugar)} + .5\text{ (gelatin)} + .5\text{ (vanilla)}] = 85$ pounds

(b) Subtract the weight of the condensed milk from the weight of the milk products. The difference represents the weight of the milk and cream in 100 pounds of mix.

$85 - (.60 \times 26.34)^{3} = 69.2$ pounds

(c) Subtract the pounds of fat in the condensed milk from the pounds of fat in the mix. The difference is the pounds of fat that must be added in the cream and milk:

$.08 \times 26.34 = 2.11$ pounds fat in the condensed milk
9 (total fat needed) $- 2.11 = 6.89$ pounds fat to be supplied by the milk and cream

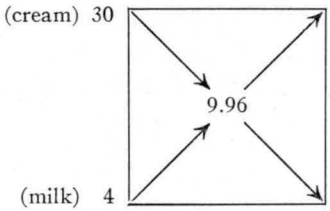
(d) Determine by the square method of standardization (or formula) the amount of cream and milk needed to supply the fat and necessary bulk:

$$\frac{6.89\text{ (fat in milk and cream)}}{69.2\text{ (weight of milk and cream)}} \times 100$$
$= 9.96$ percent fat in milk and cream mixture

[1]This product contains 40 percent sugar or 10.536 pounds, which amount must be deducted from the 14 pounds calculated as the amount needed.

[2]s.s. = serum solids.

[3]Only 60 percent of the sweetened condensed milk can be figured as a "milk product."



5.96 = parts cream

20.04 = parts milk

26.00 = total parts

Since 69.2 pounds (or parts) are needed, the necessary amounts of milk and cream are calculated:

$$5.96 : 26 = x : 69.20$$
$$26x = 5.96 \times 69.20$$
$$x = 15.86 \text{ pounds of cream needed}$$
$$69.2 - 15.86 = 53.34 \text{ pounds of milk needed}$$

(4) To find the amount of each of the ingredients needed for 1000 pounds of mix:

	Pounds	
$3.46 \times 10 =$	34.6	sugar
$.5 \times 10 =$	5.0	gelatin
$.5 \times 10 =$	5.0	vanilla
$26.34 \times 10 =$	263.4	condensed milk
$15.86 \times 10 =$	158.6	cream
$53.34 \times 10 =$	533.4	milk
	1000.0	mix

Proof

(1) 1000 pounds of mix should contain:

	Pounds	
$1000 \times .09 =$	90	fat
$1000 \times .11 =$	110	serum solids
$1000 \times .14 =$	140	sugar

(2) 1000 pounds of mix does contain:

		Pounds	
(a)	$263.4 \times .08 =$	21.07	fat in condensed milk
	$158.6 \times .30 =$	47.58	fat in cream
	$533.4 \times .04 =$	21.34	fat in milk
		89.99	(90) fat in mix
(b)	$263.4 \times .20 =$	52.68	serum solids in condensed milk
	$158.6 \times .064 =$	10.15	serum solids in cream
	$533.4 \times .088 =$	46.94	serum solids in milk
		109.77	(110) serum solids in mix
(c)	$263.4 \times .40 =$	105.36	sugar in condensed milk
		34.60	sugar to be added
		139.96	(140) sugar in mix

Algebraic Method

Problem

Desired: 100 pounds of mix containing 10 percent fat, 34 percent total solids, 12 percent sugar, .5 percent gelatin, using 3 pounds of water for the gelatin.

Available: 40-percent cream, 3.5-percent milk, 8-percent unsweetened condensed milk. Serum solids are present in the cream to the extent of 5.6 percent, in the milk 9 percent, and in the condensed milk 20 percent.

Solution

(1) To find the amount of non-milk products needed, multiply the percentage of each desired by 100:

$.12 \times 100 = 12$ pounds sugar
$.005 \times 100 = .5$ pound gelatin

(2) To find the milk products needed:

(a) Represent each of the unknowns by a letter. Let x = cream, y = milk, and z = condensed milk, each in pounds.

(b) Form an equation representing the amount of milk products in the mix:

$$x + y + z^1 = 84.5\,[100 - (12 + 3.5)]$$

(c) Form an equation representing the amount of butterfat in the mix:

$$40x \text{ (cream)} + 3.5y \text{ (milk)} + 8z \text{ (condensed milk)} = 10 \times 100$$

(d) Eliminate one of the unknowns represented in (c) by combining it with equation (b) which has been multiplied by such a number as will result in the elimination of one of the unknowns when the two equations are combined:

$$\begin{array}{rrrl} 40x + & 3.5y + & 8z = & 1000 \\ 8x + & 8y + & 8z = & 676 \text{ (equation (b) } \times 8) \\ \hline 32x - & 4.5y & = & 324 \end{array}$$

(e) Form an equation representing the amount of serum solids in the mix:

$$5.6x \text{ (cream)} + 9y \text{ (milk)} + 20z \text{ (condensed milk)} = 1150\ (11.5 \times 100)$$

(f) Again eliminate z in equation (e) by combining this equation with equation (b) which has been multiplied by such

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[1]In case the condensed milk contains added sugar, proper allowance must be made when forming the milk products equation. In this problem if the condensed milk contained 40 percent sugar, the milk products equation would have been $x + y + 0.6z = 84.5$.

a number as will result in the elimination of z when the two equations are combined:

$$\begin{array}{rl} 5.6x + 9y + 20z = 1150 & \text{(equation (e))} \\ 20x + 20y + 20z = 1690 & \text{(equation (b) } \times 20\text{)} \\ \hline -14.4x - 11y = -540 & \end{array}$$

Changing signs: $14.4x + 11y = 540$

(g) Combine equations (d) and (f), first multiplying each equation by such a number that when the two are combined one of the unknowns will be eliminated:

$$\begin{array}{rl} 352x - 49.5y = 3564 & \text{(equation (d) } \times 11\text{)} \\ 64.8x + 49.5y = 2430 & \text{(equation (f) } \times 4.5\text{)} \\ \hline 416.8x = 5994 & \end{array}$$

(h) Solve for the remaining unknown, x:

$$\frac{5994}{416.8} = 14.38 \text{ pounds of cream needed}$$

(i) Substitute the value of x in either equation (d) or (f) and solve for the remaining unknown:

$$\begin{aligned} 207.07 + 11y &= 540 \text{ (equation (f))} \\ 11y &= 540 - 207.07 \\ y &= 30.27 \text{ pounds milk needed} \end{aligned}$$

(j) Substitute values of x and y in equation (b) and solve for the remaining unknown:

$$\begin{aligned} 14.38 + 30.27 + z &= 84.5 \\ z &= 84.5 - 44.65 \\ z &= 39.85 \text{ pounds condensed needed} \end{aligned}$$

Proof

(1) 100 pounds of mix should contain:

	Pounds
$100 \times .10$	= 10 fat
$100 \times .115$	= 11.5 serum solids

(2) 100 pounds of mix does contain:

	Pounds	
(a) $14.38 \times .40$	= 5.7520	fat in cream
$30.27 \times .035$	= 1.0595	fat in milk
$39.85 \times .080$	= 3.1880	fat in condensed milk
	9.9995	(10) fat in mix
(b) $14.38 \times .056$	= .8053	serum solids in cream
$30.27 \times .09$	= 2.7243	serum solids in milk
$39.85 \times .20$	= 7.9700	serum solids in condensed milk
	11.4996	(11.5) serum solids in mix

Tabular Method[1]

By means of Table 2 and the formulas that follow, it is possible to calculate the amount of milk products needed to prepare a mix of any given composition. In the table will be found the condensed milk portion of the formulas for 64 basic mixes. In each case it is assumed that the milk products total 83 pounds, which with the necessary sugar, flavoring material, and gelatin solution will make 100 pounds of mix testing 8 percent fat and 8 percent serum solids. The table includes the necessary corrections for the amounts of unsweetened condensed milk[2] to be used when it is desired to increase the percentage of solids or change the percentage of milk products. This makes it possible to use the table to determine the amount of condensed milk needed in a mix of any desired composition or proportion of milk products.

An attempt was made to include in the table all the more common unsweetened condensed milk products, which are as follows:

(1) Condensed skim milk ranging in serum solids from 16 thru 30 percent with 1 percent interval. The fat in this product was disregarded.

(2) Condensed whole milk with 7, 8, and 9 percent fat and the same range of serum solids as in the above group.

(3) Powdered skim milk containing 1 percent fat and 94, 95, 96, and 97 percent serum solids.

Problem

Desired: 1000 pounds of mix 86 percent of which is made up of milk products, the formula to be: fat, 12 percent; serum solids, 10 percent; sugar, 13 percent; vanilla, .5 percent; gelatin, .5 percent.

Available: 40-percent cream, 4-percent milk, 8-percent unsweetened condensed milk having 25 percent serum solids.

Solution

(1) Find the amount of condensed milk needed:

(a) The condensed milk portion of the basic mix, using the condensed milk containing 8 percent fat and 25 percent serum solids (see Table 2, Mix 40) is 7.18 pounds.

(b) For a mix containing 12 percent fat, add 2.12 (4 × .53) pounds of condensed milk. And for a mix containing 10 percent serum solids, add 12 (2 × 6) pounds to the amount of condensed milk called for in the basic mix:

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[1]All of the calculations given in the table are on a basis of 100 pounds of mix.
[2]For the use of sweetened condensed milk see the example on page 21.

TABLE 2.—AMOUNTS OF CONDENSED MILK NEEDED TO STANDARDIZE 100 POUNDS OF BASIC MIX USING 64 DIFFERENT COMPOSITIONS, AND CORRECTIONS NECESSARY WHEN CHANGING THE BASIC MIX

(Basic mix: milk products, 83 percent; fat, 8 percent; serum solids, 8 percent)

Mix No.	Composition of condensed milk used		Amount of condensed milk needed	For each 1 percent change in the basic mix add or subtract the following amounts of condensed milk		
	Fat	Serum solids		For 1 percent increase in fat, *add*—	For 1 percent increase in serum solids, *add*—	For 1 percent increase in milk products, *subtract*—
	perct.	*perct.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*
1	0	16	17.21	1.28	14.38	1.31
2	0	17	15.05	1.11	12.48	1.15
3	0	18	13.36	1.00	11.21	1.01
4	0	19	12.01	.90	10.06	.90
5	0	20	10.92	.81	9.15	.83
6	0	21	10.01	.74	8.35	.76
7	0	22	9.23	.69	7.78	.70
8	0	23	8.57	.64	7.27	.64
9	0	24	8.00	.59	6.67	.61
10	0	25	7.49	.56	6.28	.56
11	0	26	7.06	.52	5.90	.54
12	0	27	6.66	.50	5.58	.51
13	0	28	6.30	.48	5.22	.47
14	0	29	5.99	.45	4.99	.45
15	0	30	5.71	.42	4.82	.44
16	7	16	15.79	1.18	13.22	1.20
17	7	17	13.95	1.04	11.67	1.06
18	7	18	12.49	.93	10.45	.95
19	7	19	11.31	.84	9.46	.86
20	7	20	10.33	.77	8.61	.78
21	7	21	9.51	.70	7.96	.72
22	7	22	8.81	.65	7.37	.67
23	7	23	8.20	.61	6.87	.62
24	7	24	7.67	.58	6.43	.58
25	7	25	7.21	.54	6.04	.55
26	7	26	6.80	.51	5.69	.51
27	7	27	6.44	.47	5.38	.49
28	7	28	6.11	.45	5.11	.47
29	7	29	5.81	.43	4.86	.44
30	7	30	5.54	.41	4.64	.42
31	8	16	15.61	1.16	13.06	1.18
32	8	17	13.83	1.00	11.53	1.07
33	8	18	12.37	.93	10.36	.93
34	8	19	11.21	.83	9.39	.85
35	8	20	10.25	.77	8.58	.78
36	8	21	9.44	.70	7.90	.71
37	8	22	8.75	.65	7.32	.66
38	8	23	8.15	.61	6.82	.62
39	8	24	7.63	.57	6.39	.58
40	8	25	7.18	.53	6.00	.54
41	8	26	6.77	.50	5.66	.51
42	8	27	6.41	.47	5.35	.49
43	8	28	6.08	.45	5.08	.46
44	8	29	5.79	.42	4.84	.44
45	8	30	5.52	.41	4.61	.42
46	9	16	15.37	1.21	12.97	1.11
47	9	17	13.62	1.06	11.48	.99
48	9	18	12.23	.94	10.29	.90

TABLE 2.—*Concluded*

Mix No.	Composition of condensed milk used		Amount of condensed milk needed	For each 1 percent change in the basic mix add or subtract the following amounts of condensed milk		
	Fat	Serum solids		For 1 percent increase in fat, *add*—	For 1 percent increase in serum solids, *add*—	For 1 percent increase in milk products, *subtract*—
	perct.	*perct.*	*lbs.*	*lbs.*	*lbs.*	*lbs.*
49	9	19	11.09	.86	9.34	.81
50	9	20	10.15	.78	8.54	.75
51	9	21	9.36	.71	7.86	.69
52	9	22	8.68	.66	7.29	.64
53	9	23	8.09	.61	6.80	.60
54	9	24	7.58	.57	6.36	.56
55	9	25	7.12	.55	5.99	.53
56	9	26	6.72	.51	5.65	.50
57	9	27	6.37	.48	5.34	.48
58	9	28	6.04	.46	5.08	.45
59	9	29	5.75	.44	4.83	.43
60	9	30	5.48	.42	5.61	.40
61	1	94	1.41	.10	1.17	.10
62	1	95	1.39	.10	1.16	.10
63	1	96	1.37	.10	1.15	.09
64	1	97	1.36	.10	1.13	.09

	Pounds
Condensed milk in basic mix	7.18
Correction for 4 percent increase in fat	2.12
Correction for 2 percent increase in serum solids	12.00
Total	21.30

(c) Since the milk products are to form 86 percent of the entire mix, correct for the additional 3 percent by subtracting 1.62 ($3 \times .54$) pounds from the total:

	Pounds
Correction for increase in proportion of milk products	1.62
Condensed milk needed (21.30 − 1.62)	19.68

(2) To find the percentage of fat in the milk and cream mixture:

$$\text{Fat in cream and milk mixture} = \frac{\text{pounds fat in mix} - \text{pounds fat in condensed}}{\text{pounds milk products in mix} - \text{pounds condensed}} \times 100$$

$$= \frac{12 - 1.574}{86 - 19.68} \times 100 = 15.72 \text{ percent}$$

(3) To find the amount of cream needed for 100 pounds mix:

$$\text{Cream} = \frac{\%\text{ fat in cream and milk mixture} - \%\text{ fat in milk}}{\%\text{ fat in cream} - \%\text{ fat in milk}} \times \text{desired weight of milk and cream}$$

$$\text{Cream} = \frac{15.72 - 4}{40 - 4} \times 66.32 = 21.59 \text{ pounds}$$

(4) To find the amount of milk needed for 100 pounds mix, subtract the combined weight of the cream and condensed milk from the weight of all the milk products:

$$86 - (19.68 + 21.59) = 44.73 \text{ pounds}$$

(5) To find the amount of non-milk products needed for 100 pounds mix:

$.005 \times 100 = .5$ pound each of gelatin and vanilla
$.13 \times 100 = 13$ pounds sugar

(6) Since the above amounts are all calculated on the basis of 100 pounds of basic mix, multiply each ingredient by 10 to find amounts necessary for 1000 pounds of mix:

	Pounds
$19.68 \times 10 =$	196.8 condensed milk
$21.59 \times 10 =$	215.9 40-percent cream
$44.73 \times 10 =$	447.3 4-percent milk
$.50 \times 10 =$	5.0 vanilla
$.50 \times 10 =$	5.0 gelatin
$13.00 \times 10 =$	130.0 sugar
	1000.0 mix

Proof

(1) 1000 pounds of mix should contain:

	Pounds
$1000 \times .12 =$	120 fat
$1000 \times .10 =$	100 serum solids

(2) 1000 pounds of mix does contain:

(a)

	Pounds
$215.9 \times .40 =$	86.36 fat in cream
$447.3 \times .04 =$	17.89 fat in milk
$196.8 \times .08 =$	15.74 fat in condensed milk
	119.99 (120) fat in mix

(b)

$215.9 \times .055 =$	11.874 serum solids in cream
$447.3 \times .087 =$	38.915 serum solids in milk
$196.8 \times .25 =$	49.20 serum solids in condensed milk
	99.989 (100) serum solids in mix

Possible Variations of Tabular Method

When the Composition of the Condensed Milk Product Does Not Fall Within the Range of Table 2

In this case, if extreme accuracy is desired, the exact amount of condensed milk needed for the basic mix and the corrections for fat, for serum solids, and for the proportion of milk products can

easily be calculated from the values of these factors given for the next highest and next lowest testing condensed-milk product.

For example, suppose that the condensed milk product on hand contains 7.5 percent fat and 22 percent serum solids. The amount of condensed milk needed for the basic mix is determined as follows:

	Pounds
Condensed milk containing 8 percent fat and 22 percent serum solids needed for basic mix	8.75
Condensed milk containing 7 percent fat and 22 percent serum solids needed for basic mix	8.81
Difference	.06
One-half this difference	.03
Adding this correction to the amount of 8-percent condensed milk needed	8.78

The correction for a difference in the percentage of serum solids or in the proportion of milk products can be calculated in the same way.

Using a Sweetened Condensed Milk Product

In case the condensed milk contains sugar, the percentage of milk products must be raised. The steps are illustrated in the solution of the following problem.

Problem

Desired: A mix containing 12 percent fat, 12 percent serum solids and 12 percent sugar.

Available: 40 percent cream, 4-percent milk, 8-percent condensed milk containing 25 percent serum solids and 40 percent sugar.

Solution

(1) From Table 2 calculate the amount of unsweetened condensed milk that would be needed:

	Pounds
Basic mix requires	7.18
Correction for additional 4 percent fat	2.12
Correction for additional 4 percent serum solids	24.00
Total unsweetened condensed milk	33.30

(2) Multiply the amount of condensed milk that must be deducted from the basic mix (Table 2) in order to raise the milk product 1 percent by the percentage of sugar in the sweetened condensed milk:

$$.54 \times .4 = .216$$

(3) Multiply the weight of the unsweetened condensed milk needed by the percentage of sugar in the sweetened condensed milk:

$$33.30 \times .4 = 13.32$$

(4) Divide the result obtained in (3) by 1 plus the result obtained in (2):

$$\frac{13.32}{1.216} = 10.95 \text{ percent milk products that must be added}$$

(5) According to Table 2 (Mix 40) there will be a .54 pound decrease in the milk products for each 1 percent increase over the basic mix as given; therefore,

	Pounds condensed milk
83.00 percent milk products in mix requires	33.30
10.95 percent increase (10.95 × −.54)	−5.91
93.95 percent milk products in mix requires	27.39

(6) To calculate the pounds of sugar needed for 100 pounds of mix when the 27.39 pounds of sweetened condensed milk used contains 40 percent sugar:

27.39 × .4 = 10.96 pounds of sugar in condensed milk
12 − 10.96 = 1.04 pounds of sugar needed for 100 pounds of mix containing 12 percent sugar

(7) The finished product will contain the following ingredients:

	Pounds
Cream (by formula)	19.83
Milk (by formula)	46.73
Condensed milk	27.39
Sugar	1.04
Gelatin	.50
Vanilla	.50
Water for gelatin	4.00
Total weight of mix	99.99 (100)

Proof

(1) 100 pounds of mix should contain:

Pounds

100 × .12 = 12 fat
100 × .12 = 12 serum solids
100 × .12 = 12 sugar

(2) 100 pounds of mix does contain:

Pounds

(a) 19.83 × .40 = 7.9320 fat in cream
46.73 × .04 = 1.8692 fat in milk
27.39 × .08 = 2.1912 fat in condensed milk
11.9924 (12) fat in mix

(b) 19.83 × .055 = 1.0907 serum solids in cream
46.73 × .087 = 4.0655 serum solids in milk
27.39 × .25 = 6.8475 serum solids in condensed milk
12.0037 (12) serum solids in mix

(c) 27.39 × .40 = 10.956 sugar in condensed milk
1.040 sugar to be added
11.996 (12) sugar in mix

Using More Than One Condensed Milk Product

In case more than one condensed milk product is used, proceed as in the following example:

Problem

Desired: 1000 pounds of mix containing 11 percent fat, 12 percent serum solids, .5 percent gelatin, .5 percent vanilla, and 12 percent sugar.

Available: 40-percent cream, 4-percent milk, unsweetened condensed skim milk containing 27 percent serum solids, and 20 pounds of skim-milk powder containing 94 percent serum solids.

Solution

(1) In case powder alone is to be used, the following milk products will be needed (calculated from Table 2 and the formulas): Cream, 218.2 pounds, milk, 547.9 pounds, and skim-milk powder, 63.9 pounds.

(2) The amount of powder (20 pounds) that it is desired to use represents 31.3 percent of the total amount of powder that would be needed if it were to be used as the only condensed milk product. Therefore 31.3 percent of each of the above amounts will be needed for the 20 pounds of powder: cream, 68.30 pounds; milk, 171.49 pounds.

(3) The remainder of the milk products will represent 100 − 31.3 = 68.7 percent of the amount in pounds of each product that would be needed if 27-percent condensed milk were to be used as the only condensed milk product: cream, 169.76; milk, 191.05; and condensed milk, 209.40.

(4) Adding the amounts of milk and cream that go with the 20 pounds of milk powder to those that go with the 209.40 pounds of 27-percent condensed milk as calculated above, the total of each needed is as follows:

Milk: 171.49 + 191.05 = 362.54 pounds
Cream: 68.30 + 169.76 = 238.06 pounds

(5) Adding the non-milk products needed for a 1000-pound batch of the composition indicated, the completed formula is as follows:

	Pounds
Cream	238.06
Milk	362.54
Skim-milk powder	20.00
Condensed milk	209.40
Sugar	120.00
Gelatin	5.00
Vanilla	5.00
Water	40.00
Total mix	1000.00

Proof

(1) 1000 pounds of mix should contain:

Pounds

1000 × .11 = 110 fat
1000 × .12 = 120 serum solids

(2) 1000 pounds of mix does contain:

Pounds

(a) 238.06 × .40 = 95.2240 fat in cream
362.54 × .04 = 14.5016 fat in milk
20.00 × .01 = .2000 fat in powder
109.9256 (110) fat in mix

(b) 238.06 × .055 = 13.0933 serum solids in cream
362.54 × .087 = 31.5410 serum solids in milk
209.40 × .27 = 56.5380 serum solids in condensed milk
20 × .94 = 18.8000 serum solids in powder
119.9723 (120) serum solids in mix

III—RESTANDARDIZING AN OFF-BATCH

A finished batch of ice-cream mix does not always have either the right ratio or the right concentration of solids, and it becomes necessary to restandardize it by adding the amounts of milk products, or non-milk products, or both, that are needed to correct the wrong proportions. How may the amounts to be added be determined?

In order to determine whether or not the *solids* are of the proper ratio and concentration, it is necessary to determine the percentage of fat and total solids in the mix[1] and then, from the weight of the finished product and the weight of the sugar and gelatin used, to calculate the percentage of *serum solids* by taking the difference.

There are three types of restandardization problems: (1) those encountered when the solids are low; (2) those encountered when the solids are high; and (3) those encountered when a part of the solids are low and a part are high.

[1]The Mojonnier milk tester is most commonly used for determining the percentage of fat and total solids in the mix.

When the Solids Are Low

Problem

Given: 1000 pounds of mix containing 9 percent fat, 9 percent serum solids, 13.5 percent sugar, .45 percent gelatin.

Desired: A mix containing 10 percent fat, 10 percent serum solids, 15 percent sugar, and .5 percent gelatin.

Available: 40-percent cream, unsweetened condensed milk containing 8 percent fat and 25 percent serum solids.

Arithmetical Solution

(1) Determine the number of pounds each solid is short:

	Percentage short	*Pounds of mix*	*Pounds short*
Fat	.1	1000	10.0
Serum solids	.1	1000	10.0
Sugar	1.5	1000	15.0
Gelatin	.05	1000	.5

(2) Assume that 300 pounds are to be added to correct the mix. Using the arithmetical[1] procedure described on page 12, determine the amount of each of the different products needed to satisfy the requirements stated:

(a) To find the amount of non-milk solids needed:

(.15 × 300) 45 + 15 = 60 pounds sugar
(.005 × 300) 1.5 + .5 = 2 pounds gelatin

(b) To find the amount of condensed milk needed:

The mix serum must contain 40 pounds serum solids
10 (shortage in original mix) + 30 (added mix) = 40

300 − [40 (fat) + 60 (sugar) + 2 (gelatin)] = 198 pounds serum in mix to be added

198 × .092 = 18.22 pounds normal serum solids present

40 − 18.22 = 21.78 pounds serum solids that must be added in addition to the normal serum solids

92 = percentage of serum in condensed milk

92 × .092 = 8.46 percentage of normal serum solids present in condensed milk

25 − 8.46 = 16.54 percentage of serum solids present in condensed milk in addition to the percentage of normal serum solids

$$\frac{21.78}{16.54} \times 100 = 131.68 \text{ pounds condensed milk that must be added}$$

[1]The tabular method may be used if desired.

(c) To find the amount of 40-percent cream and skim milk needed:

$131.68 \times .08 = 10.534$ pounds fat in condensed milk

$40 - 10.534 = 29.466$ pounds fat to be supplied by the 40-percent cream

$\frac{29.466}{40} \times 100 = 73.67$ pounds of 40-percent cream needed

The amount of skim milk needed is found by difference:

	Pounds
Condensed milk to add	131.68
Cream to add	73.67
Sugar to add	60.00
Gelatin to add	2.00
Total to add (excluding skim milk)	267.35

Skim milk needed = 300 − 267.35 = 32.65 pounds.

Proof

(1) 1300 pounds of mix should contain:

	Pounds
$1300 \times .10$	= 130 fat
$1300 \times .10$	= 130 serum solids
$1300 \times .15$	= 195 sugar
$1300 \times .005$	= 6.5 gelatin

(2) 1300 pounds of mix does contain:

Pounds

(a) $1000 \times .09 = 90.000$ fat in unstandardized mix
$73.67 \times .40 = 29.468$ fat in cream
$131.68 \times .08 = 10.534$ fat in condensed milk
130.002 fat in finished mix

(b) $1000 \times .09 = 90.000$ serum solids in unstandardized mix
$73.67 \times .055 = 4.052$ serum solids in cream
$32.65 \times .092 = 3.004$ serum solids in skim milk
$131.68 \times .25 = 32.920$ serum solids in condensed milk
129.976 (130) serum solids in finished mix

(c) $1000 \times .13 = 135$ sugar in unstandardized mix
60 sugar to be added
195 sugar in finished mix

(d) $1000 \times .045 = 4.5$ gelatin in unstandardized mix
2.0 gelatin to be added
6.5 gelatin in finished mix

Algebraic Solution

(1) Following the procedure described under the algebraic method, page 15, formulate equations for fat, serum solids, and milk

products. The three unknowns are: x, amount of cream needed; y, amount of condensed milk needed; and z, weight of finished product.

1. Fat equation $(9 \times 1000) + 40x + 8y = 10z$
2. Serum solids equation $(9 \times 1000) + 5.5x + 25y = 10z$
3. Milk products equation $(.8605 \times 1000) + x + y = .845z$

(2) Combine and recombine the equations until only one unknown remains. Solve for that unknown and determine the others by substitution:

(a) $9000 + 40x + 8y = 10z$ (equation 1)
$6884 + 8x + 8y = 6.76z$ (equation 3, $\times 8$)
$2116 + 32x = 3.24z$
$32x - 3.24z = -2116$

(b) $9000 + 5.5x + 25y = 10z$ (equation 2)
$21512.5 + 25x + 25y = 21.125z$ (equation 3, $\times 25$)
$-12512.5 - 19.5x = -11.125z$
$-19.5x + 11.125z = 12512.5$

(c) $-32x + 18.26z = 20533.01$ (equation (b) $\times 1.641$)
$32x - 3.24z = -2116.00$ (equation (a))
$15.02z = 18417.01$
$z = 1226.16$ pounds in finished product

(d) Substituting value of z in equation (a):
$32x = 3972.76 - 2116$
$x = 58.02$ pounds of cream needed

(e) Substituting value of x and z in equation 1:
$8y = 12261.6 - 9000 - 2320.8$
$y = 117.6$ pounds of condensed milk needed

(f) $1226.16 \times .15 = 183.92$ pounds sugar needed
$1000 \times .135 = 135.00$ pounds sugar present
48.92 pounds sugar to add

(g) $1226.16 \times .005 = 6.13$ pounds gelatin needed
$1000 \times .0045 = 4.50$ pounds gelatin present
1.63 pounds gelatin to add

Proof

(1) Total weight should be 1226.16 pounds. The above calculations call for:

	Pounds
Unstandardized mix	1000.00
Cream to be added	58.02
Condensed milk to be added	117.60
Sugar to be added	48.92
Gelatin to be added	1.63
Total mix	1226.17

(2) 1226.16 pounds of mix should contain:

	Pounds
1226.16 × .10	= 122.616 fat
1226.16 × .10	= 122.616 serum solids
1226.16 × .15	= 183.920 sugar
1226.16 × .005	= 6.131 gelatin

(3) 1226.17 pounds of mix does contain:

Pounds

(a) 1000 × .09 = 90.00 fat in unstandardized mix
58.02 × .40 = 23.208 fat in cream
117.60 × .08 = 9.408 fat in condensed milk
122.616 fat in finished mix

(b) 1000 × .09 = 90.00 serum solids in unstandardized mix
58.02 × .055 = 3.191 serum solids in cream
117.60 × .25 = 29.400 serum solids in condensed milk
122.591 serum solids in finished mix

(c) 1000 × .135 = 135.00 sugar in unstandardized mix
48.92 sugar to be added
183.92 sugar in finished mix

(d) 1000 × .045 = 4.5 gelatin in unstandardized mix
1.63 gelatin to be added
6.13 gelatin in finished mix

When the Solids Are High

Problem

Given: 1000 pounds of mix containing 12 percent fat, 14 percent serum solids, 16 percent sugar, and .65 percent gelatin.

Desired: The ingredients needed to make a mix containing 10 percent fat, 10 percent serum solids, 15 percent sugar, and .5 percent gelatin.

Arithmetical Solution

(1) Determine the total weight that the mix will need to be from the weight of the solid present in the greatest proportional excess in the given mix. To determine the solid:

(a) Determine the ratio of fat to each of the other solids in the desired mix by dividing the percentage of each of the solids by the percentage of fat:

	Fat	*Serum solids*	*Sugar*	*Gelatin*
10 |	10	10	15	.5
	1	1	1.5	.05

(b) Multiply the above ratios by the percentage of fat in the unstandardized mix:

Fat	*Serum solids*	*Sugar*	*Gelatin*
1	1	1.5	.05
	× 12		
12	12	18	.6

The given mix contains:

12	14	16	.65

By inspection it is seen that the serum solids are present in greatest proportional excess. Therefore, cream[1] must be added.

(c) Multiply the weight of the mix by the percentage of serum solids.

$$1000 \times .14 = 140 \text{ pounds}$$

(d) Divide the result by the desired percentage of serum solids and multiply by 100:

$$\frac{140}{10} \times 100 = 1400 \text{ pounds}$$

The total mix then will weigh 1400 pounds, which means that 400 pounds will have to be added to obtain a mix of the desired standard.

(2) Calculate the amounts of fat, sugar, and gelatin that will be needed in the 400 pounds of added mix:

$$(1400 \times .10) - (1000 \times .12) = 20 \text{ pounds fat}$$
$$(1400 \times .15) - (1000 \times .16) = 50 \text{ pounds sugar}$$
$$(1400 \times .005) - (1000 \times .0065) = .5 \text{ pounds gelatin}$$

(3) The question that now confronts us is how we may add the necessary amount of fat without adding serum solids, no more of which are desired.

In case the milk product being used to supply the necessary fat contains only a small amount of serum solids, such as 40-percent cream, its content of serum solids may be disregarded. The only step left in that case is to figure the pounds of 40-percent cream that will be needed to supply the necessary fat:

$$(20 \div 40) \times 100 = 50 \text{ pounds of 40-percent cream necessary}$$

[1]If either of the non-milk solids were present in highest proportional excess, both cream and condensed milk would be needed. If the fat were the high solid, then condensed milk would have to be added, whereas if the serum solids had the highest proportional excess, then cream would be needed. In some cases it may be necessary to add water to obtain the necessary bulk.

However, occasions may arise in which the milk product would add appreciable amounts of the unneeded solid. In this case further computations and adjustments are necessary. We may take as an example the use of 20-percent cream containing 7.4 percent serum solids. The steps follow:

(a) In order to maintain the desired ratio between the milk solids, 7.4 percent fat must be considered as unavailable for standardizing. The amount of cream needed is then:

$$\frac{20}{20 - 7.4} \times 100 = 158.73 \text{ pounds}$$

(b) Calculating the serum solids added by the cream:

$158.73 \times .074 = 11.74$ pounds

(c) Calculating the amount of additional mix that must be added to take care of the extra serum solids in the cream:

$$\frac{11.74}{10} \times 100 = 117.4 \text{ pounds}$$

(d) Total weight of sugar needed:

$50 + (117.4 \times .15) = 67.61$ pounds

(e) Total weight of gelatin needed:

$.5 + (117.4 \times .005) = 1.087$ pounds

(f) Total weight of products added:

$158.73 + 67.61 + 1.09 = 227.43$ pounds

(g) Water needed for bulk:

$(400 + 117.40) - 227.43 = 289.97$ pounds

Proof

(1) Total weight of mix should be 1517.4 pounds. The above calculations call for:

	Pounds
Unstandardized mix	1000.00
Cream to be added	158.73
Sugar to be added	67.61
Gelatin to be added	1.09
Water to be added	289.97
Total mix	1517.40

(2) 1517.4 pounds of mix should contain:

	Pounds
$1517.4 \times .10$	= 151.74 fat
$1517.4 \times .10$	= 151.74 serum solids
$1517.4 \times .15$	= 227.61 sugar
$1517.4 \times .005$	= 7.59 gelatin

(3) 1517.4 pounds of mix does contain:

Pounds

(a) 1000 × .12 = 120.000 fat in unstandardized mix
158.73 × .20 = 31.746 fat in cream
151.746 fat in finished mix

(b) 1000 × .14 = 140.00 serum solids in unstandardized mix
158.73 × .074 = 11.746 serum solids in cream
151.746 serum solids in finished mix

(c) 1000 × .16 = 160.00 sugar in unstandardized mix
67.61 sugar to be added
227.61 sugar in finished mix

(d) 1000 × .0065 = 6.50 gelatin in unstandardized mix
1.09 gelatin to be added
7.59 gelatin in finished mix

Algebraic Solution

(1) As explained under the arithmetical solution, page 28, determine which solid is present in greatest proportional excess. If it is a non-milk solid its weight will determine the finished weight of the mix, otherwise the finished weight will be one of the unknowns. Since in this case the serum solids are present in greatest proportional excess, the unknowns are x, finished weight, and y, weight of cream needed (20% cream is available).

(2) Formulate equations and solve for the unknowns as explained on page 5:

(a) $10x = 20y + (12 \times 1000)$ (fat equation)
$10x = 7.4y + (14 \times 1000)$ (serum solids equation)

(b) $20y + 12000 = 7.4y + 14000$
$12.6y = 2000$
$y = 158.73$ pounds cream needed

(c) $x = 1517.4$ pounds finished mix

(d) 1517.4 × .15 = 227.61 pounds sugar needed
1000 × .16 = 160.00 pounds sugar in mix
67.61 pounds sugar to add

(e) 1517.4 × .005 = 7.587 pounds gelatin needed
1000 × .0065 = 6.50 pounds gelatin in mix
1.087 pounds gelatin to add

(f) The finished product will then consist of 1000 pounds of unstandardized mix plus cream, sugar, and gelatin as found above with water (289.97 pounds) to make the desired bulk. For proof see arithmetical solution, p. 14.

When Part of Solids Are Low and Part Are High

Problem

Given: 1000 pounds of mix containing 11.5 percent fat, 9 percent serum solids, 11 percent sugar, and .55 percent gelatin.

Desired: A mix containing 10 percent fat, 11 percent serum solids, 13 percent sugar, and .5 percent gelatin.

Available: 30-percent cream, skim milk (9.2 percent serum solids), and 8-percent unsweetened condensed milk (25 percent serum solids).

Arithmetical Solution

(1) As explained on page 28, determine the total weight the mix will need to be from the weight of the solid present in the mix in the greatest proportional excess:

	Fat	*Serum solids*	*Sugar*	*Gelatin*
10 |	10	11	13	5
Desired ratio....	1	1.1	1.3	.05
		× 11.5		
	11.5	12.65	14.95	.575
Unstandardized mix contains.....	11.5	9	11	.550

It is apparent that the fat is present in highest proportional excess. Calculating the amounts of other solids needed to bring them into the desired proportion with the fat:

$$1000 \times .115 = 115 \text{ pounds of fat on hand}$$

$$\frac{115}{10} \times 100 = 1150 \text{ pounds mix needed to equalize the fat}$$

(2) Subtract the weight of the unstandardized mix from the weight of the mix necessary to equalize the fat as obtained in (1). The result represents the amount of bulk that must be added:

$$1150 - 1000 = 150 \text{ pounds bulk to be added}$$

(3) The short solids are found as follows:

	Fat	*Serum solids*	*Sugar*	*Gelatin*
Pounds solids needed.....	115	126.5	149.5	5.75
Subtract pounds solids on hand.....	115	90.0	110.0	5.50
Pounds to be added.....	0	36.5	39.5	.25

(4) Determine the amount of milk products and non-milk products that would be needed to supply the solids calculated in (3), in the bulk calculated in (2). It is evident that it would be impossible to supply the short solids in a total weight of 150 pounds when using the available milk products. Therefore add 400 pounds instead. This amount is arbitrarily chosen and can be reduced to a minimum, as will be shown later.

(a) The 400 pounds of mix to be added must contain, besides the solids calculated in (3), the following, in order to supply the solids in the additional 250 pounds:

$250 \times .10 = 25$ pounds fat
$250 \times .11 = 27.5$ pounds serum solids
$250 \times .13 = 32.5$ pounds sugar
$250 \times .005 = 1.25$ pounds gelatin

(b) Total solids needed in additional mix are:

$0.0 + 25 = 25$ pounds fat
$36.5 + 27.5 = 64$ pounds serum solids
$39.5 + 32.5 = 72$ pounds sugar
$.25 + 1.25 = 1.50$ pounds gelatin

(c) To find the amount of condensed milk needed:

$400 - (72 + 25 + 1.5) = 301.500$ pounds serum in portion to be added
$301.50 \times .092 = 27.74$ pounds normal serum solids
$(100 - 8) \times .092 = 8.46$ percent normal serum solids in condensed milk
$\frac{64 - 27.74}{25 - 8.46} \times 100 = 219.22$ pounds condensed milk needed

(d) To find the amount of 30-percent cream needed:

$219.22 \times .08 = 17.54$ pounds fat in condensed milk
$25 - 17.54 = 7.46$ pounds fat needed in cream
$\frac{7.46}{30} \times 100 = 24.87$ pounds 30-percent cream needed

(e) To find the skim milk needed:

$301.50 - (219.22 - 17.54) = 99.82$
$99.82 - (24.87 - 7.46) = 82.41$ pounds skim milk needed

(f) The products needed to standardize the mix are then:

	Pounds
Condensed milk	219.22
30-percent cream	24.87
Skim milk	82.41
Sugar	72.00
Gelatin	1.50

Proof

(1) Total weight of mix should be 1400 pounds. The above calculations call for:

	Pounds
Unstandardized mix	1000.00
Condensed milk to be added	219.22
Cream to be added	24.87
Skim milk to be added	82.41
Sugar to be added	72.00
Gelatin to be added	1.50
Total mix	1400.00

(2) 1400 pounds of mix should contain:

	Pounds	
1400 × .10	= 140.00	fat
1400 × .11	= 154.00	serum solids
1400 × .13	= 182.00	sugar
1400 × .005	= 7.00	gelatin

(3) 1400 pounds of mix does contain:

		Pounds	
(a)	1000 × .115	= 115.00	fat in unstandardized mix
	24.87 × .30	= 7.46	fat in cream
	219.22 × .08	= 17.54	fat in condensed milk
		140.00	fat in finished mix
(b)	1000 × .09	= 90.00	serum solids in unstandardized mix
	24.87 × .0644	= 1.60	serum solids in cream
	219.22 × .25	= 54.81	serum solids in condensed milk
	82.41 × .092	= 7.58	serum solids in skim milk
		153.99	serum solids in finished mix
(c)	1000 × .11	= 110	sugar in unstandardized mix
		72	sugar to be added
		182	sugar in finished mix
(d)	1000 × .0055	= 5.5	gelatin in unstandardized mix
		1.5	gelatin to be added
		7.0	gelatin in finished mix

It is evident that if an amount somewhat less than 400 pounds had been added, there would have been no need of adding either the 30-percent cream or skim milk, since the fat solids were in excess. Therefore, to find the *minimum* amounts of condensed milk, sugar, and gelatin that need to be added, proceed as follows:

(1) The fat and serum solids added by the cream and skim milk are as follows:

By 30-percent cream	7.46 pounds fat	1.60 pounds serum solids
By skim milk		7.58 pounds serum solids
	7.46 pounds fat	9.18 pounds serum solids

(2) The desired ratio of fat to serum solids is such that 7.46 pounds fat require 8.206 pounds serum solids. If, therefore, both the cream and skim milk were removed from the above mixture, there would be a shortage of .974 pounds (9.180 − 8.206) of serum solids. Enough condensed milk must be added to supply this shortage, proper allowance being made for the serum solids in the condensed milk that are *unavailable* for standardizing:

$$\frac{.974}{(25 - 8.8)} \times 100 = 6.01 \text{ pounds condensed milk to add}$$

(3) Since less milk solids are being added, less sugar and gelatin will be needed. The reduced amount of fat multiplied by the fat-sugar ratio will give the pounds of sugar to be deducted from the calculated amount of sugar needed:

$$(7.46 - .48) \times 1.3 = 9.07$$

(4) In the same way the correction for gelatin is obtained:

$$(7.46 - .48) \times .05 = .35$$

(5) Therefore the corrected list of products to be added is:

219.22	+ 6.01 =	225.23 pounds condensed milk
72.00	− 9.07 =	62.93 pounds sugar
1.5	− .35 =	1.15 pounds gelatin
	Total	289.31 pounds

(6) The correct weight for the finished mix can be calculated from the weight of any of the solids. If calculated on the sugar basis, the procedure would be as follows:

Weight of sugar in original mix	110.00 pounds
Weight of sugar added	62.93 pounds
Total sugar	172.93 pounds

$$\frac{172.93}{13} \times 100 = 1330.23 \text{ pounds finished weight}$$

1330.23 − (1000 + 289.31) = 40.92 pounds shortage in bulk, which should be made up by the addition of water

Algebraic Solution

(1) As explained under algebraic solution, page 31, determine whether cream or condensed milk or both[1] need to be added. The unknowns are x, condensed milk needed, and y, finished weight.

[1]There may be times when there is only a slight excess of one milk solid and a large deficiency of the other, in which case there will be three unknowns: finished weight, weight of cream, and weight of condensed milk.

(2) Formulate equations and solve for the unknowns, as explained on page 15:

(a) $(11.5 \times 1000) + 8y = 10x$ (fat equation)
$(9.0 \times 1000) + 25y = 11x$ (serum solids equation)

(b) $12650.00 + 8.8y = 11x$ (fat equation $\times$ 1.1)
$9000.00 + 25y = 11x$
$3650.00 - 16.2y = 0$
$16.2y = 3650$
$y = 225.31$ pounds condensed milk needed

(c) $11x = 9000 + 5632.75$, $x = 1330.25$ weight of finished mix

(d) $1330.25 \times .13 = 172.93$ pounds sugar needed
$1000 \times .11 = 110.00$ pounds sugar in mix
62.93 pounds sugar to add

(e) $1330.25 \times .005 = 6.65$ pounds gelatin needed
$1000 \times .0055 = 5.50$ pounds in mix
1.15 pounds gelatin to add

(f) Additions: condensed milk (225.31), sugar (62.93), gelatin (1.15)..... 289.39 pounds
Original mix..... 1000.00 pounds
1289.39 pounds
$1330.25 - 1289.39 = 40.86$ pounds water needed for bulk

Proof

(1) 1330.25 pounds mix should contain:

		Pounds
$1330.25 \times .10$	=	133.025 fat
$1330.25 \times .11$	=	146.3275 serum solids
$1330.25 \times .13$	=	172.93 sugar
$1330.25 \times .005$	=	6.65 gelatin

(2) 1330.25 pounds mix does contain:

		Pounds	
(a) $1000.00 \times .115$	=	115.00	fat from unstandardized mix
$225.31 \times .08$	=	18.025	fat from condensed milk
		133.025	fat in finished mix
(b) $1000 \times .092$	=	90.000	serum solids in unstandardized mix
$225.31 \times .25$	=	56.3275	serum solids in condensed milk
		146.3275	serum solids in finished mix
(c) $1000 \times .11$	=	110.00	sugar in unstandardized mix
		62.93	sugar to be added
		172.93	sugar in finished mix
(d) $1000 \times .0055$	=	5.50	gelatin in unstandardized mix
		1.15	gelatin to be added
		6.65	gelatin in finished mix